For this problem set, you may find it useful to consult Ken Rosen’s textbook *Discrete Math and Its Applications*.

1. Give the contrapositive of the following statement. “If every bird flies, then there is a hungry cat.”

Answer:

* If there is not a hungry cat, then no bird flies.

1. A proposition is a statement that can be true or false but not both. Let A, B, and C be propositions. Let denote logical AND, let denote logical OR, and let denote logical NOT. Argue that if is true, then must be true as well.

Answer:

* is true if all possible values in its domain map to “True” in its range.
* The table below maps each possibility of A, B and C through each of the terms of the expression
* In each case the resulting value is true
* Therefore , or if is true, then must be true as well.

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| A | B |  | C |  |  |  |  |  |
| T | T | F | T | T | T | T | T | T |
| T | T | F | F | T | F | F | T | T |
| T | F | T | T | T | T | T | T | T |
| T | F | T | F | T | T | T | T | T |
| F | T | F | T | T | T | T | T | T |
| F | T | F | F | T | F | F | F | T |
| F | F | T | T | F | T | F | T | T |
| F | F | T | F | F | T | F | F | T |

1. We use the notation to indicate that A implies B. This new proposition is true except when A is true and B is false. We write when either both A and B are true or both are false. Argue that if and only if and .

Answer:

* Two expressions are equivalent if they map to the same values in a range for any values in their domain
* All possible values of the domain are given in the truth table
* If the values in the range of each expression match for every possible combination of values in the domain, they are equivalent
* has the truth table shown below, demonstrating that is true except when A is true and B is false.
* Similarly is true except when B is true and A is false (see table)
* is defined to be true when either both A and B are true or both are false (see table).
* and can be represented as and is also true only when either both A and B are true or both are false (see table).
* Hence if and only if and .

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| A | B |  |  |  |  |
| T | T | T | T | T | T |
| T | F | F | T | F | F |
| F | T | T | F | F | F |
| F | F | T | T | T | T |

1. We will use the notation to indicate the number of elements in the set or its *cardinality*, e.g. is the number of elements in the set A. Consider four sets A,B,C,D such that the intersection of any three is empty. Use the inclusion-exclusion to give an expression for without using any union ( symbols.

Answer:

1. State the formal definition of , and show that the function is .

Answer:

Definition: Let f and g be functions from the set of integers or the set of real numbers to the set of real numbers. We say that f (x) is O(g(x)) if there are constants C and k such that

|f (x)| ≤ C|g(x)|

whenever x > k. [Source: Rosen, *Discrete Mathematics and Its Applications, 7e*, pp. 205]

* Let C = 2, k = 5, and g(x) = n
* |f (x)| ≤ 2|g(x)| for x > 5 and diverging more as x grows (see graph)
* Further, the limit as n approaches infinity of f(x) = n
* Let A be a set. We use the notation to indicate the power set of A, which consist of all subsets of A. For example, if , then . Consider and use an inductive argument to show that

for all positive integers n.

* Prove that the set of all languages that have a bounded maximum string length is countable.